

7. A. A. Bochkarev, E. G. Velikanov, A. K. Rebrov, R. G. Sharafutdinov, and V. N. Yarygin, "Low density gasdynamic equipment," in: *Experimental Methods in Rarefied Gasdynamics* [in Russian], S. S. Kutateladze (ed.), Izd. Teplofiz. Inst., Novosibirsk (1974).
8. A. V. Kosov, A. K. Rebrov, and R. G. Sharafutdinov, "Ejector action of high speed low density jet," in: *Nonequilibrium Processes in Rarefied Gas Flows* [in Russian], Inst. Teplofiz. Novosibirsk (1977).
9. D. Golomb and R. E. Good, "Dimers, clusters, and condensation in free jets," *J. Chem. Phys.*, 37, No. 2 (1972).

FLOW IN THE HYPERSONIC BOUNDARY LAYER ON A FINITE TRIANGULAR  
WING IN THE PRESENCE OF AN ANGLE OF ATTACK

G. N. Dudin

UDC 533.6.011.55

The investigation of three-dimensional viscous gas flows at hypersonic flight velocities is of importance to the determination of the aerodynamic characteristics. It has been established in numerous experimental studies (see [1], for instance) that the nature of the flow in the boundary layer on flat delta wings depends substantially on the magnitude of the hypersonic interaction parameter  $\chi = M_\infty^2 Re^{1/2}$ , where  $M_\infty$  is the free stream Mach number, and  $Re = \rho_\infty U_\infty L / \mu_0$  is the Reynolds number determined from values of the density and velocity in the unperturbed stream, the wing length, and the viscosity coefficient at the stagnation temperature. Two limiting flow regimes can be examined here. In the weak interaction regime ( $\chi \sim 0.1$ ) even for a small angle of attack vortices [2] which drift downstream, occur within the boundary layer on the leeward side of a delta wing, and their interaction with the body surface results in an increase in friction and heat flux. In the strong viscous interaction regime ( $\chi \geq 1$ ) [3], at least up to moderate angles of attack, attached flow is realized over the whole wing. However, it should be noted that the nature of the flow in a region near the apex of the wing is identical in both cases since the parameter is  $\chi \geq 1$  there (the Reynolds number should be calculated relative to the length of the domain under consideration). The flow around a thin delta plate in the strong viscous interaction regime has been investigated theoretically at zero angle of attack in [4-7]. Examination of the flow around a semiinfinite triangular plate permits reduction of the boundary value problem to a self-similar problem, for whose solution the methods developed for two-dimensional problems are applicable. However, the system remains three-dimensional in the consideration of the flow around a delta wing at an angle of attack in the strong viscous interaction regime. A solution is obtained in [8] for the system of Navier-Stokes equations near a semiinfinite delta wing at an angle of attack, but an assumption is made here that the gradients in the radial direction are much less than in the others, and the boundary-value problem is reduced to a self-similar problem.

1. The flow of a hypersonic stream of viscous gas around a finite delta wing at an angle of attack  $\alpha^\circ$  is considered in this paper under the assumption that the perturbed part of the flow contains a stream inviscid in a first approximation, which is described by the hypersonic theory of small perturbations [9] and the viscous boundary layer. It is assumed that the angle of attack is small ( $\alpha^\circ < \tau$ ) and such that the assumption of the hypersonic theory of small perturbations is always satisfied

$$M_\infty(\tau \pm \alpha^\circ) \geq O(1), \quad (1.1)$$

where  $\tau = (s/Re)^{1/4}$  is the characteristic dimensionless boundary-layer thickness ( $s = \tan \beta$ ,  $\beta$  is the half-angle at the wing apex). The plus sign in (1.1) corresponds to flow around the lower (windward) wing surface, and the minus sign around the upper (leeward) surface. The Cartesian coordinate system whose origin is at the apex of the delta wing (the  $x^\circ$  axis is directed along the axis of symmetry, the  $z^\circ$  axis along the span, and the  $y^\circ$  axis along the normal to the wing surface) is presented in Fig. 1. It is assumed that boundary-layer interaction with the external hypersonic stream is strong ( $\chi > 1$ ) on the whole wing. The

---

Moscow. Translated from *Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki*, No. 3, pp. 108-113, May-June, 1983. Original article submitted May 14, 1982.

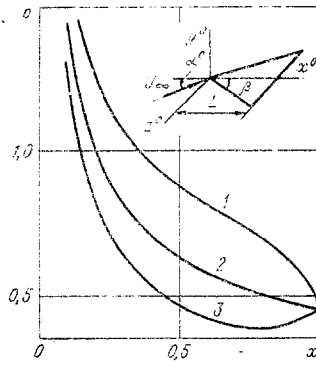


Fig. 1

solution of the complete boundary-value problem includes taking account of the flow in the wake which is formed behind the wing [10]; however, in this paper, in order not to consider this flow, the boundary condition is given on the trailing edge of the delta wing. It should here be kept in mind that a series expansion of the solution in the neighborhood of the leading edge contains an arbitrary function for the flow around a plate that is not cold since the flow is precritical [10], and hence, it is also necessary to give a function for the selection of a unique solution of the boundary-value problem on the trailing edge.

In conformity with the usual estimates for the boundary layer in a hypersonic stream [9], the following dimensionless variables are introduced:  $uU_\infty$ ,  $wU_\infty$ ,  $v_W \tau s^{-1} U_\infty$ , projections of the total velocity on the axes  $x^0 = xL$ ,  $z^0 = zL$ ,  $y^0 = y\tau L$ ,  $\rho \tau^2 \rho_\infty U_\infty^2$ , pressure;  $gU_\infty^2/2$ , stagnation enthalpy;  $\mu\mu_0$ , dynamic coefficient of viscosity; and  $\delta_e \tau L$ , boundary-layer displacement thickness. Furthermore, the case of a linear dependence of the viscosity on the temperature is considered. Substitution of the variables mentioned into the Navier-Stokes equations and passing to the limit  $Re \rightarrow \infty$  result in the spatial boundary-layer equations which have the following form in the A. A. Dorodnitsyn variables

$$\begin{aligned} su \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial \lambda} + w \frac{\partial u}{\partial z} &= -\frac{s}{\rho} \frac{\partial p}{\partial x} + \frac{\partial}{\partial \lambda} \left( \mu \rho \frac{\partial u}{\partial \lambda} \right), \\ su \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial \lambda} + w \frac{\partial w}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial z} + \frac{\partial}{\partial \lambda} \left( \mu \rho \frac{\partial w}{\partial \lambda} \right), \\ su \frac{\partial g}{\partial x} + v \frac{\partial g}{\partial \lambda} + w \frac{\partial g}{\partial z} &= \frac{\partial}{\partial \lambda} \left\{ \mu \rho \left[ \frac{1}{\sigma} \frac{\partial g}{\partial \lambda} - \frac{1-\sigma}{\sigma} \frac{\partial (u^2 + w^2)}{\partial \lambda} \right] \right\}, \\ s \frac{\partial u}{\partial x} + \frac{\partial v}{\partial \lambda} + \frac{\partial w}{\partial z} &= 0, \quad \rho = \frac{2\gamma}{\gamma-1} \frac{p}{g - u^2 - w^2}, \quad \mu = g - u^2 - w^2, \end{aligned} \quad (1.2)$$

$$\lambda = \int_0^y \rho dy, \quad v = \rho v_W + w \frac{\partial \lambda}{\partial z} + su \frac{\partial \lambda}{\partial x},$$

where  $\sigma$  is the Prandtl number, and  $\gamma$  is the ratio of the specific heats. The boundary conditions for  $|z| \leq x$ ,  $0 \leq x \leq 1$ :  $u = v = w = 0$ ,  $g = g_W$  ( $\lambda = 0$ );  $u \rightarrow 1$ ,  $w \rightarrow 0$ ,  $g \rightarrow 1$  ( $\lambda \rightarrow \infty$ ). To solve this system of equations it is necessary to know the pressure distribution which is not given and should be determined during the process of solving the problem (1.2) in conjunction with the equations for the external inviscid flow described by the hypersonic theory of small perturbations. The simultaneous solution of these two systems of equations for the viscous and inviscid flows is fraught with great difficulties. Since the flow around a wing with the span  $s \sim O(1)$  is considered in this paper, and the theory of strips [9] is valid for the external inviscid flow, then the approximate formula for a tangent wedge [9] can be used to determine the pressure, for example, in a form valid for  $M_\infty(\tau \pm \alpha^\circ) \gg 1$  ( $\alpha^\circ = \alpha\tau$ ,  $0 \leq d < 1$ ):

$$p = \frac{\gamma+1}{2} \left( \frac{\partial \delta_e}{\partial x} \pm \alpha \right)^2, \quad (1.3)$$

where  $\delta_e$  is the boundary-layer displacement thickness determined by the expression [4]

$$\delta_e = \frac{\gamma-1}{2\gamma p} \int_0^\infty (g - u^2 - w^2) d\lambda. \quad (1.4)$$

2. For the numerical solution of the boundary-value problem (1.2)-(1.4), the singularities in the behavior of the flow function should be taken into account in the neighborhood of the delta wing apex. To do this, the following variables are introduced

$$x = x, \quad z = x\theta, \quad \lambda = x^{1/4}\lambda^*, \quad p = x^{-1/2}p^*(x, \theta), \quad (2.1)$$

$$\rho = x^{-1/2}\rho^*(x, \lambda^*, \theta), \quad \delta_e = x^{3/4}\delta_e^*(x, \theta), \quad v = x^{-3/4}\left(v^* - xus \frac{\partial \lambda}{\partial x}\right).$$

Without presenting the complete system of equations in the variables (2.1), let us note that the expression for the pressure (1.3) takes the form

$$p^*(x, \theta) = \frac{\gamma+1}{2} \left( \frac{3}{4} \delta_e^* + x \frac{\partial \delta_e^*}{\partial x} - \theta \frac{\partial \delta_e^*}{\partial \theta} \pm \alpha x^{1/4} \right)^2.$$

For  $\alpha \neq 0$  and  $x > 0$  on the wing surface, the pressure turns out to be a function of  $x$  and  $\theta$  because of the presence of the term  $\alpha x^{1/4}$  in this expression, while the flow in the boundary layer depends on three variables  $x$ ,  $\theta$  and  $\lambda$ . Therefore, even for a semiinfinite wing, in the presence of an angle of attack the system of equations (1.2)-(1.4) in the variables (2.1) remains three-dimensional, in contrast to the case of flow around a wing at a zero angle of attack when the system of three-dimensional boundary-layer equations (1.2)-(1.4) reduces to a system dependent just on the two independent variables  $\theta$  and  $\lambda^*$  [7]. To take account of the singularities in the behavior of the flow function in the strong viscous interaction regime in the neighborhood of the leading edges of the delta wing ( $\theta = \pm 1$ ), we introduce the variables

$$\lambda^* = \sqrt{\frac{2\gamma}{\gamma-1} (1-\theta^2)^{1/2} \eta}, \quad p^* = (1-\theta^2)^{-1/2} p_0(x, \theta),$$

$$\delta_e^* = (1-\theta^2)^{3/4} \Delta_e(x, \theta), \quad v^* = \left[ v_0 \frac{p_0}{1-\theta^2} - (w - su\theta) \frac{\partial \eta}{\partial \theta} \right] \sqrt{\frac{2\gamma}{\gamma-1} (1-\theta^2)^{1/2}}. \quad (2.2)$$

Taking (2.1) and (2.2) into account, system (1.2)-(1.4) takes the form

$$sux \frac{1-\theta^2}{p_0} \frac{\partial f}{\partial x} + (w - su\theta) \frac{1-\theta^2}{p_0} \frac{\partial f}{\partial \theta} + v_0 \frac{\partial f}{\partial \eta} = G,$$

$$f = \begin{pmatrix} u \\ w \\ g \end{pmatrix}, \quad G = \begin{cases} -s \frac{\gamma-1}{2\gamma p_0} (g - u^2 - w^2) \left[ (1-\theta^2) \left( \frac{\partial \ln p_0}{\partial \ln x} - \frac{1}{2} \right) - \theta \left( \theta + \frac{1-\theta^2}{p_0} \frac{\partial p_0}{\partial \theta} \right) \right] + \frac{\partial^2 u}{\partial \eta^2}, \\ -\frac{\gamma-1}{2\gamma p_0} (g - u^2 - w^2) \left( \theta + \frac{1-\theta^2}{p_0} \frac{\partial p_0}{\partial \theta} \right) + \frac{\partial^2 w}{\partial \eta^2}, \\ \frac{1}{\sigma} \frac{\partial^2 g}{\partial \eta^2} - \frac{1-\sigma}{\sigma} \frac{\partial^2 (u^2 + w^2)}{\partial \eta^2}, \end{cases} \quad (2.3)$$

$$\frac{\partial v_0}{\partial \eta} = (w - su\theta) \frac{\theta}{2p_0} \frac{1-\theta^2}{p_0} \left( sx \frac{\partial u}{\partial x} - s\theta \frac{\partial u}{\partial \theta} + \frac{\partial w}{\partial \theta} + \frac{su}{4} \right),$$

$$p_0 = \frac{\gamma+1}{2} \left( \frac{3}{4} (1-\theta^2) \Delta_e + x(1-\theta^2) \frac{\partial \Delta_e}{\partial x} - \theta (1-\theta^2) \frac{\partial \Delta_e}{\partial \theta} - \frac{3}{2} \theta \Delta_e \right) \pm \alpha x^{1/4} (1-\theta^2)^{1/4},$$

$$\Delta_e = \sqrt{\frac{\gamma-1}{2\gamma} \frac{1}{p_0}} \int_0^\infty (g - u^2 - w^2) d\eta.$$

For  $|\theta| \ll 1$  and  $0 \leq x \leq 1$  the boundary conditions are:

$$u = w = v_0 = 0, \quad g = g_W (\eta = 0); \quad u \rightarrow 1, w \rightarrow 0, \quad g \rightarrow 1 (\eta \rightarrow \infty).$$

The system of partial differential equations (2.3) describes the flow in a three-dimensional boundary layer on a delta wing of finite length at an angle of attack in the strong viscous interaction regime. It should be noted that at the wing apex ( $x = 0$ ) the terms containing the variable  $x$  in system (2.3) drop out and the boundary-value problem turns out to be dependent on just two independent variables  $\theta$  and  $\eta$ . For the values  $\theta = \pm 1$  of the transverse coordinate at the wing leading edges, system (2.3) degenerates into a system of ordinary differential equations. The domain of integration of the system (2.3) is a rectangular parallelepiped. To solve the boundary-value problem (2.3) it is first necessary to solve the system of ordinary differential equations on the wing leading edges, then

by using these solutions as boundary conditions, to solve the system of partial differential equations dependent on two variables and describing the flow at the delta wing apex. Finally, taking account of the boundary condition on the wing trailing edge, the pressure distribution, say, and also taking account of the solution obtained at the wing apex and at its leading edges, the system of three-dimensional boundary-layer equations (2.3) is solved. Let us note that for a given pressure distribution  $p_0^1(\theta)$  on the wing trailing edge, the displacement thickness  $\Delta_e(x, \theta)$  obtained because of solving the complete boundary value problem should satisfy the relationship

$$\frac{\gamma + 1}{2} \left\{ \frac{3}{4} (1 - \theta^2) \Delta_e + (1 - \theta^2) \frac{\partial \Delta_e}{\partial x} - \theta \left[ (1 - \theta^2) \frac{\partial \Delta_e}{\partial \theta} - \frac{3}{2} \theta \Delta_e \right] \pm \alpha (1 - \theta^2)^{1/4} \right\}^2 = p_0^1(\theta) \quad (2.4)$$

for the value  $x = 1$  of the longitudinal coordinate. In solving the total boundary-value problem including the wake, the unique solution should be selected from the condition of compliance with certain relationships presented in [10] on the "sonic" surface on which the transition into the post-critical regime occurs.

To solve the systems of differential equations mentioned, the method of finite differences is used. Second-order difference schemes in  $\Delta\eta$  and first or second order in  $\Delta\theta$  and  $\Delta x$  are used to approximate the equations, where  $\Delta\eta$ ,  $\Delta\theta$ ,  $\Delta x$  are the spacings along the coordinates  $\eta$ ,  $\theta$ ,  $x$ . The derivatives with respect to the coordinates  $\theta$  and  $x$  in the equations are approximated in the difference scheme with the sign preceding them taken into account. The systems of difference equations for the functions  $u$ ,  $w$ , and  $g$  are solved successively by the method of scalar factorization, one after the other. The system of nonlinear equations is here replaced by a linear system of difference equations in each iteration, where the relaxation values from the preceding iteration are used for the linearization [11]. The difference analog of the continuity equation reduces to a first-order difference equation which was always approximated to second-order accuracy.

The iteration process is the following. For a certain approximate pressure distribution  $p_0(x, \theta)$ , which agrees with the given  $p_0^1(\theta)$  at  $x = 1$ , the motion and energy equations of (2.3) are solved. Since central differences for  $x > 0$  were used to take account of upstream transmission of perturbations to approximate the quantities  $\partial p_0 / \partial x$ , then this pressure gradient is not known for  $x = 1$  on the last layer and is selected from the condition of satisfying the relationship (2.4) during the solution of the complete boundary-value problem (2.3). The values obtained for the flow functions  $u$ ,  $w$ , and  $g$  and for the pressure  $p_0(x, \theta)$  are used to determine the displacement thickness  $\Delta_e$ . Then from the tangent wedge formula a new pressure is found, and the iteration process is continued until the required accuracy is achieved. As numerical computations have shown, both the flow functions  $u$ ,  $w$ , and  $g$  and the pressure  $p_0$  must be relaxed for the stability of the difference boundary-value problem, where the relaxation coefficients turned out to be on the order of  $\varphi_1 = 0.2-0.5$ , respectively, for  $u$ ,  $w$ , and  $g$ , and  $\varphi_2 = 0.02-0.05$  for  $p_0$ . It should be noted that the pressure for the value  $x = 1$  was naturally not relaxed. The iterations were terminated when the maximum difference in two successive iterations for the quantities  $p_0(x, \theta)$  (as the most slowly convergent quantity) and the difference between the pressure given on the trailing edge  $p_0^1(\theta)$  and the pressure calculated there  $p_0(x = 1, \theta)$ , became less than  $10^{-4}$ . Here 300-400 iterations were required for this.

3. As an illustration, the flow around a delta wing on whose trailing edge the pressure identically equals the pressure corresponding to the flow around a semiinfinite delta wing with the value of the coordinate  $x = 1$  and zero angle of attack [7] is considered in this paper.

In the numerical computations it was assumed that  $s = 2$  (the sweepback angle  $\sim 27^\circ$ ),  $\gamma = 1.4$ ,  $\sigma = 0.71$ ;  $g_W = 0.5$ , and  $\alpha = 0, 0.3$ . Results of a computation for the pressure along the axis of symmetry of the wing ( $z = 0$ ) are represented in Fig. 1. The values of  $p$  with  $\alpha = -0.3$  correspond to a pressure distribution on the upper wing surface (curve 3) and with  $\alpha = 0.3$  to the lower surface (curve 1). Curve 2 ( $\alpha = 0$ ) corresponds to a flow at zero angle of attack. As should have been expected, the pressure values on the wing leeward surface is considerably greater than on the windward surface; thus, for  $x = 0.5$  the pressure on the lower side is almost twice as great as on the upper. As numerical computations showed, a change in the magnitude of the pressure at the trailing edge exerts influence on the upstream flow at  $\approx 30-40\%$  of the wing chord. Therefore, the pressure distribution on the wing surface from the apex to the value  $x = 0.6$  of the longitudinal coordinate depends only on the angle of

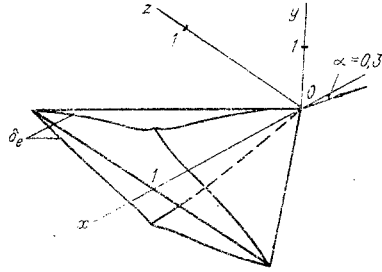


Fig. 2

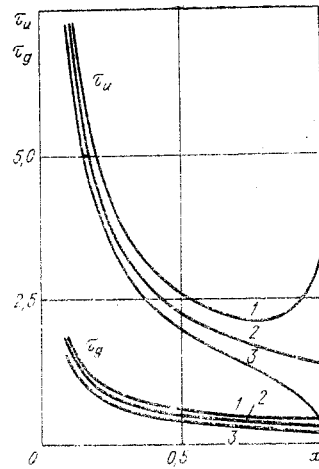


Fig. 3

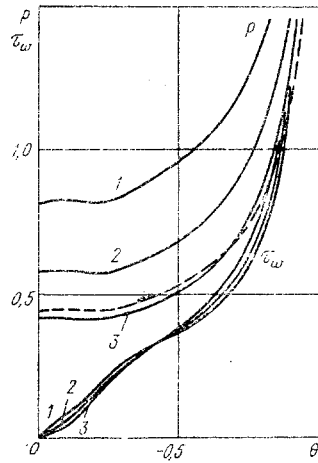


Fig. 4

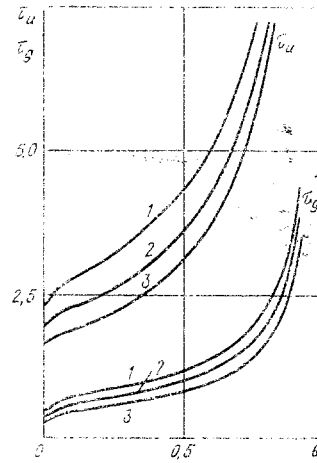


Fig. 5

attack (and also the quantities  $s$ ,  $\gamma$ ,  $\sigma$  and  $g_w$ ), but is independent of the pressure distribution given at the trailing edge if this given pressure is not so large as to cause boundary-layer separation at the wing. The distribution of the boundary-layer displacement thickness  $\delta_e(x, z)$  on a wing over which the flow is at the angle of attack  $\alpha = 0.3$  is represented in Fig. 2. As was also noted in experimental investigations of flows in a strong viscous interaction regime [3], a significant growth in  $\delta_e(x, z)$  occurs on the leeward side and its diminution on the windward side. Especially strong changes in the displacement thickness occur in the neighborhood of the plane of symmetry. Results of a computation of the friction stress coefficient in the longitudinal direction  $\tau_u = \partial u / \partial y|_W$  and of the thermal flux  $\tau_g = \partial g / \partial y|_W$  on the wing surface along the axis of symmetry  $z = 0$  are presented in Fig. 3. Values of the thermal flux and the friction stress coefficient on the windward side of the wing ( $\alpha = 0.3$ ) considerably exceed their magnitude on the leeward side of the wing ( $\alpha = -0.3$ ). A sharp growth in the quantity  $\tau_u$ , related to acceleration of the stream, is observed on the wing lower surface in the neighborhood of the trailing edge. The weak influence of the magnitude of the pressure given on the trailing edge on the thermal flux distribution should also be noted.

Distributions of  $p$ ,  $\tau_u$ ,  $\tau_g$ , and the friction stress coefficient in the transverse direction  $\tau_w = \partial w / \partial y|_W$  over the wing space are represented in Figs. 4 and 5 for the value  $x = 0.6$  of the longitudinal coordinate. This value of the coordinate  $x$  is chosen from the conditions noted above. The dashed line in Fig. 4 denotes the value of the pressure  $p(x=1)$  at the trailing edge at which all the computations presented in this paper were performed. It must be noted that the influence of the magnitude of the angle of attack on the friction stress coefficient in the transverse direction  $\tau_w$  is comparatively weak, at least for the value  $|\theta| > 0.2$ . However, near the plane of symmetry  $|\theta| \leq 0.1$ , the quantity  $\tau_w$  on the windward side considerably exceeds its value on the leeward side.

LITERATURE CITED

1. A. H. Whitehead, Jr., J. N. Hefner, and D. M. Rao, "Lee-surface vortex effects over configurations in hypersonic flow," AIAA Paper No. 72-77 (1972).
2. D. M. Rao and A. H. Whitehead, Jr., "Lee-side vortices on delta wings at hypersonic speeds," AIAA J., 10, No. 11 (1972).
3. E. J. Cross, Jr., and W. L. Hankey, "Investigation of leeward side of a delta wing at hypersonic speeds," AIAA Paper No. 68-675 (1968).
4. M. D. Ladyzhenskii, "On spatial hypersonic flow around thin wings," Prikl. Mat. Mekh., 28, No. 5 (1964).
5. I. G. Kozlova and V. V. Mikhailov, "On strong viscous interaction on triangular and sliding wings," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 6 (1970).
6. V. Ya. Neiland, "On the theory of hypersonic flow interaction with the boundary layer for two- and three-dimensional separation flows. Pt. 2. Two-dimensional flows and a delta wing," Uch. Zap. TsAGI, 5, No. 3 (1974).
7. G. N. Dudin, "On a boundary-layer computation on a triangular plate in the strong viscous interaction regime," Uch. Zap., Ts. AGI, 9, No. 5 (1978).
8. G. S. Bluford, Jr., "Numerical solution of the supersonic and hypersonic viscous flow around thin delta wings," AIAA Paper No. 78-1136 (1978).
9. W. D. Hayes and R. F. Probstein (eds.) Hypersonic Flow Theory, Academic Press (1967).
10. V. Ya. Neiland, "On the theory of hypersonic flow interaction with the boundary layer for two- and three-dimensional separation flows. Pt. 1. Three-dimensional flows," Uch. Zap., TsAGI, 5, No. 2 (1974).
11. Yu. D. Shevelev, Three-Dimensional Problems of Laminar Boundary Layer Theory [in Russian], Nauka, Moscow (1977).

STABILITY OF A PLANE CRYSTALLIZATION FRONT MOVING AT  
CONSTANT VELOCITY

L. G. Badratinova

UDC 532.78:536.421.4

1. In an  $x, y, z$  coordinate system coupled to a plane unperturbed front (the  $x$  axis is directed into the melt and the  $y, z$  axes are along the interfacial surface), the crystallization process of a dilute binary alloy is described by the equations

$$\begin{aligned} \text{for } x > f(y, z, t) \quad \partial T_1 / \partial t + \mathbf{v}_1 \cdot \nabla T_1 = \chi_1 \Delta T_1, \\ \partial c_1 / \partial t + \mathbf{v}_1 \cdot \nabla c_1 = D \Delta c_1, \quad \nabla \cdot \mathbf{v}_1 = 0, \\ \partial \mathbf{v}_1 / \partial t + (\mathbf{v}_1 \cdot \nabla) \mathbf{v}_1 = \nu \Delta \mathbf{v}_1 - \nabla p / \rho_1 + \mathbf{g}, \quad \mathbf{g} = (-g, 0, 0); \end{aligned} \quad (1.1)$$

$$\text{for } x < f(y, z, t) \quad \partial T_2 / \partial t - V_2 (\partial T_2 / \partial x) = \chi_2 \Delta T_2 \quad (1.2)$$

with local phase equilibrium conditions [1]

$$x = f(y, z, t) \quad T_1 = T_2 = mc_1 + T_0 + T_0 \gamma K, \quad (1.3)$$

no tangential component of the melt velocity on the front, and continuity of the energy and mass fluxes [2] of both melt components during passage through the interface  $x = f(y, z, t)$ :

$$\begin{aligned} (\kappa_2 \nabla T_2 - \kappa_1 \nabla T_1) \mathbf{n} = -\rho_1 \Lambda (\mathbf{v}_1 - \mathbf{U}) \mathbf{n}, \\ \mathbf{v}_1 \cdot \boldsymbol{\tau} = 0, \quad \rho_1 (\mathbf{v}_1 - \mathbf{U}) \mathbf{n} = \rho_2 (\mathbf{v}_2 - \mathbf{U}) \mathbf{n}, \\ D \rho_1 \rho_2^{-1} \nabla c_1 \mathbf{n} = (1 - k) c_1 (\mathbf{v}_2 - \mathbf{U}) \mathbf{n}. \end{aligned} \quad (1.4)$$

Here  $\mathbf{v}_1$ ,  $p$ , and  $c_1$  are the melt velocity, pressure, and impurity concentration (measured in weight fractions),  $T_j$  ( $j = 1, 2$ ) are the temperatures of the medium. The subscript